Derivation of the 3 point frequency formula

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11/5/08

If we assume uniform sampling and we have 3 consecutive samples, y_0, y_1, and y_2. Then if we assume they are consecutive samples of a sinusoid, then we may write them as:

$$y_0 = A\cos(\varphi - \theta) = A\cos(\varphi)\cos(\theta) - A\sin(\varphi)\sin(\theta) \qquad [1]$$

$$y_1 = A\cos(\varphi) \qquad [2]$$

$$y_2 = A\cos(\varphi + \theta) = A\cos(\varphi)\cos(\theta) + A\sin(\varphi)\sin(\theta) \qquad [3]$$

And here A is the amplitude and φ is an arbitrary phase and θ is the angular step per sample.

Since our angular step, $\theta = 2\pi \frac{f}{f_s}$, then we need to find theta to be able to back solve for f.

So let's add [1]+[3]

$$y_0 + y_2 = 2A\cos(\varphi)\cos(\theta) \qquad [4]$$

Now just divide [4] by 2 times [2] and we find:

$$\frac{y_0 + y_2}{2y_1} = \cos(\theta) \qquad [5]$$

Thus

$$\theta = \cos^{-1}\left(\frac{y_0 + y_2}{2y_1}\right) \qquad [6]$$

Finally

$$f = \frac{f_s}{2\pi}\theta = \frac{f_s}{2\pi}\cos^{-1}\left(\frac{y_0 + y_2}{2y_1}\right) \qquad [7]$$