Ghost Cancellation Reference (GCR) Details

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When one looks at the GCR details as given in the ATSC spec, it appears that one is trying to obfuscate the nature of the signal. So what I do here is take the info from the spec and work it to a simple usable form so one may synthesize the GCR frequency chirp.

The GCR signal consists of a windowed linear frequency chirp. The window function is given in the spec as:

$$W(\omega)=\int_{-\frac{\pi}{c}}^{\frac{\pi}{c}}\left[\left(\frac{1}{2}+\frac{1}{2}\cos(ct)\right)\left(\frac{1}{2\pi}\int_{-\Omega_1}^{\Omega_1}e^{j\gamma t}\,d\gamma\right)\right]e^{-j\omega t}\,dt$$

The linear frequency chirp is given in the spec by the following equation:

$$f(t)=\frac{A}{2\pi}\left\{\int_{0}^{\Omega}\left[\cos(b\omega^2)+j\sin(b\omega^2)\right]W(\omega)e^{j\omega t}\,d\omega+\int_{-\Omega}^{0}\left[\cos(b\omega^2)-j\sin(b\omega^2)\right]W(\omega)e^{j\omega t}\,d\omega\right\}$$

The constants are specified to be

constant	value	units
A	9.0	
b	110.0	
c	$\frac{\pi}{49}$	Radians/sec
Ω	$\frac{4.3}{7.16}\pi$	Radians/sec
Ω_1	$\frac{4.15}{7.16}\pi$	Radians/sec

Notes:

1. The frequencies have been scaled to assume a system sampled at 4 times color burst. That's why the division by 7.16. Basically what they did here was to multiply a frequency (in MHz) by 2pi/(4 times color burst) but then they factored out a 2, so they only needed to multiply by pi/(2 times color burst) And only 3 digits of precision are used.
2. The chirp equation as it is written in the spec. has a problem. The freq. squared term and the complex exponential terms need to rotate in opposite directions, so the integration can find a constant region based on t, and return a nonzero result. As it is written , the factors rotate the same way, so the integration returns nearly zero results for all positive time. The fix for this will be shown later in this document.

In order to actually find the numerical result, we will reduce some of the math. We will start with the window function. The inner integral may be evaluated with the following result:

$$W(\omega)=\int_{-\frac{\pi}{c}}^{\frac{\pi}{c}}\left[\left(\frac{1}{2}+\frac{1}{2}\cos(ct)\right)\left(\frac{1}{2\pi}\frac{2\sin(\Omega_1 t)}{t}\right)\right]e^{-j\omega t}dt$$

Next one may do some manipulation of the constants to yield:

$$W(\omega)=\frac{1}{2\pi}\int_{-\frac{\pi}{c}}^{\frac{\pi}{c}}\left[(1+\cos(ct))\left(\frac{\sin(\Omega_1 t)}{t}\right)\right]e^{-j\omega t}dt$$

Which we can see is a windowed sinc function. The window is a raised cosine function. Next we will exploit the fact that the raised cosine and the sinc functions are even and find thus:

$$W(\omega)=\frac{1}{2\pi}\int_{-\frac{\pi}{c}}^{\frac{\pi}{c}}\left[(1+\cos(ct))\left(\frac{\sin(\Omega_1 t)}{t}\right)\right]\cos(\omega t)dt$$

Since the integrand is a product of even functions, the integrand itself is even, so we can perform another simplification.

$$W(\omega)=\frac{1}{\pi}\int_{0}^{\frac{\pi}{c}}\left[(1+\cos(ct))\left(\frac{\sin(\Omega_1 t)}{t}\right)\right]\cos(\omega t)dt$$

Now expand out the integrand

$$W(\omega)=\frac{1}{\pi}\int_{0}^{\frac{\pi}{c}}\frac{\sin(\Omega_1 t)\cos(\omega t)+\sin(\Omega_1 t)\cos(\omega t)\cos(ct)}{t}dt$$

Next apply a couple of trig identities (for products of functions), and we find:

$$W(\omega)=\frac{1}{4\pi}\int_{0}^{\frac{\pi}{c}}\frac{2\sin(\xi t)}{t}+\frac{2\sin(\psi t)}{t}+\frac{\sin([\psi-c]t)}{t}+\frac{\sin([\psi+c]t)}{t}+\frac{\sin([\xi-c]t)}{t}+\frac{\sin([\xi+c]t)}{t}dt$$

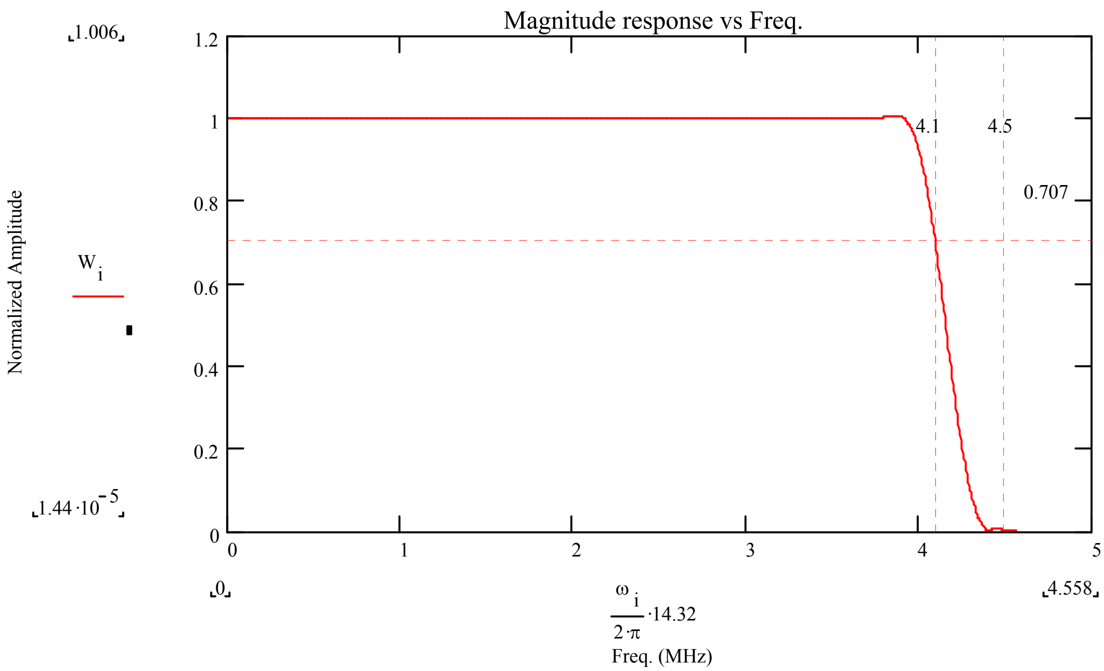
Where we define $\xi=\Omega_1-\omega$, $\psi=\Omega_1+\omega$. Now if we distribute the integration across the 6 terms, we see that we have 6 integrals. Each one is effectively a sine integral, but a simple change of variable on each one will put each one into standard form. The result is:

$$W(w)=\frac{1}{4\pi}\left[2Si\left(\frac{\pi}{c}\xi\right)+2Si\left(\frac{\pi}{c}\psi\right)+Si\left(\frac{\pi}{c}(\psi-c)\right)+Si\left(\frac{\pi}{c}(\psi+c)\right)+Si\left(\frac{\pi}{c}(\xi-c)\right)+Si\left(\frac{\pi}{c}(\xi+c)\right)\right]$$

Where we define $Si(x) \equiv \int_0^x \frac{\sin(\kappa)}{\kappa} d\kappa$

This may seem at first to complicate things, but the sine integral is a well-behaved function and can be easily calculated.

A plot of the window function is as follows:



Now we need to reduce the equation for the linear frequency chirp to usable form. So starting with the original equation from the spec, we have:

$$f(t) = \frac{A}{2\pi}\left\{\int_0^{\Omega}\left[\cos(b\omega^2) + j\sin(b\omega^2)\right]W(\omega)e^{j\omega t}d\omega + \int_{-\Omega}^{0}\left[\cos(b\omega^2) - j\sin(b\omega^2)\right]W(\omega)e^{j\omega t}d\omega\right\}$$

The 1st thing to do is to collapse the cos()+jSin() terms into complex exponentials. Thus:

$$f(t) = \frac{A}{2\pi}\left\{\int_0^{\Omega} e^{jb\omega^2}W(\omega)e^{j\omega t}d\omega + \int_{-\Omega}^{0} e^{-jb\omega^2}W(\omega)e^{j\omega t}d\omega\right\}$$

Now realizing that the window function is even, let's negate omega on the second integral. This results in:

$$f(t) = \frac{A}{2\pi}\left\{\int_0^{\Omega} e^{jb\omega^2}W(\omega)e^{j\omega t}d\omega - \int_{\Omega}^{0} e^{-jb\omega^2}W(\omega)e^{-j\omega t}d\omega\right\}$$

Next reverse the limits in the right hand integral, yielding:

$$f(t) = \frac{A}{2\pi}\left\{\int_0^\Omega e^{jb\omega^2} W(\omega) e^{j\omega t} d\omega + \int_0^\Omega e^{-jb\omega^2} W(\omega) e^{-j\omega t} d\omega\right\}$$

Now we can combine the two integrals:

$$f(t) = \frac{A}{2\pi}\int_0^\Omega \left[e^{j(\omega t + b\omega^2)} + e^{-j(\omega t + b\omega^2)}\right] W(\omega) d\omega$$

Which after applying Euler's identity becomes:

$$f(t) = \frac{A}{\pi}\int_0^\Omega \cos\left(\omega t + b\omega^2\right) W\left(\omega\right) d\omega$$

I believe the error in the spec was that the exponential rotated the wrong way. If the rotation is reversed, the following form results for the linear chirp equation:

$$f(t) = \frac{A}{\pi}\int_0^\Omega \cos\left(\omega t - b\omega^2\right) W\left(\omega\right) d\omega$$

This results in a single sign change. In the original equation this amounts to two swapped signs. Here is a plot resulting from the corrected equation. The abscissa is in units of 1/14.318MHz.