Resonator Parameter Table (Clay S. Turner)

This table list the parameters for some common and not so common 2^{nd} order discrete time sinusoidal oscillators/resonators. These may be operated as stand alone oscillators or driven as in Goertzel applications. Each oscillator/resonator has two state variables. Here they are assumed to be a and b .The oscillator frequency is determined by the step angle theta. This is the increment in angle per iteration. The tuning parameter, κ , is determined by θ . The iteration equations calculated in the given order are what you need to do per iteration to update the oscillator. If you are not driving the resonator with data, then the x_n term is simply zero. If you are using a resonator in a Goertzel application, then the real and imaginary components of the Fourier coefficient are c_k and s_k . In Goertzel apps, after driving the oscillator with all of the data, then iterate one more time with zero input to get the phase correct. The energy is given by *E*.

Resonator Parameter Table for Common Configurations						
Resonator	$\kappa =$	Iteration equations	Matrix	c_k	s_k	E
Biquad	$2\cos(\theta)$	$\tau = \kappa \cdot a - b + x_n$ $b = a$ $a = \tau$	$\begin{bmatrix} \kappa & -1 \\ 1 & 0 \end{bmatrix}$	$a - \cos(\theta) \cdot b$	$-\sin(\theta) \cdot b$	$a^2 + b^2 - \kappa \cdot a \cdot b$
Waveguide	$\cos(\theta)$	$\tau = \kappa(a + b)$ $\rho = \tau - b + x_n$ $b = \tau + a$ $a = \rho$	$\begin{bmatrix} \kappa & \kappa - 1 \\ \kappa + 1 & \kappa \end{bmatrix}$	a	$-\tan\left(\frac{\theta}{2}\right) \cdot b$	$a^2 + \frac{1-\kappa}{1+\kappa} \cdot b^2$
Magic Circle	$2\sin\left(\frac{\theta}{2}\right)$	$b = b - \kappa a$ $a = a + \kappa b + x_n$	$\begin{bmatrix} 1-\kappa^2 & \kappa \\ -\kappa & 1 \end{bmatrix}$	$a - \sin\left(\frac{\theta}{2}\right) \cdot b$	$\cos\left(\frac{\theta}{2}\right) \cdot b$	$a^2 + b^2 - \kappa \cdot a \cdot b$
Staggered type #2	$\cos(\theta)$	$\tau = \kappa b - a$ $a = b - \kappa\tau + x_n$ $b = \tau$	$\begin{bmatrix} \kappa & 1-\kappa^2 \\ -1 & \kappa \end{bmatrix}$	a	$\sin(\theta) \cdot b$	$a^2 + \left(1 - \kappa^2\right) \cdot b^2$

Coupled	$\sin(\theta)$	$\tau = \sqrt{1-\kappa^2}a + \kappa b + x_n$ $b = -\kappa a + \sqrt{1-\kappa^2}b$ $a = \tau$	$\begin{bmatrix} \sqrt{1-\kappa^2} & \kappa \\ -\kappa & \sqrt{1-\kappa^2} \end{bmatrix}$	a	b	$a^2 + b^2$
Reinsch	$2\cos(\theta) - 1$	$\tau = \kappa a + b$ $b = \tau - a$ $a = \tau + x_n$	$\begin{bmatrix} \kappa & 1 \\ \kappa - 1 & 1 \end{bmatrix}$	$a - \frac{b}{2}$	$\frac{1}{2}\cot\left(\frac{\theta}{2}\right) \cdot b$	$a^2 + \frac{b^2}{1-\kappa} - a \cdot b$
Type “A”	$4\sin^2\left(\frac{\theta}{2}\right)$	$b = a + b$ $a = a - \kappa b + x_n$	$\begin{bmatrix} 1-\kappa & -\kappa \\ 1 & 1 \end{bmatrix}$	$a + 2\sin^2\left(\frac{\theta}{2}\right) \cdot b$	$-\sin(\theta) \cdot b$	$a^2 + \kappa \cdot b(a+b)$
Type “B”	$2\cos(\theta)$	$\tau = b$ $b = \kappa b - a$ $a = \tau + x_n$	$\begin{bmatrix} 0 & 1 \\ -1 & \kappa \end{bmatrix}$	$a + \cos(\theta) \cdot b$	$\sin(\theta) \cdot b$	$a^2 + b^2 - \kappa \cdot a \cdot b$
Type “C”	$4\cos^2\left(\frac{\theta}{2}\right)$	$b = -(a+b)$ $a = -\kappa b - a + x_n$	$\begin{bmatrix} \kappa - 1 & \kappa \\ -1 & -1 \end{bmatrix}$	$a + 2\cos^2\left(\frac{\theta}{2}\right) \cdot b$	$\sin(\theta) \cdot b$	$a^2 + \kappa \cdot b(a+b)$
Type “C” Modified	$-4\sin^2\left(\frac{\theta}{2}\right)$	$b = b + a$ $a = a + \kappa b + x_n$	$\begin{bmatrix} \kappa + 1 & \kappa \\ 1 & 1 \end{bmatrix}$	$a + 2\sin^2\left(\frac{\theta}{2}\right) \cdot b$	$-\sin(\theta) \cdot b$	$a^2 - \kappa \cdot b(a+b)$