Geostationary Satellites and Astrophotography

By Clay S. Turner

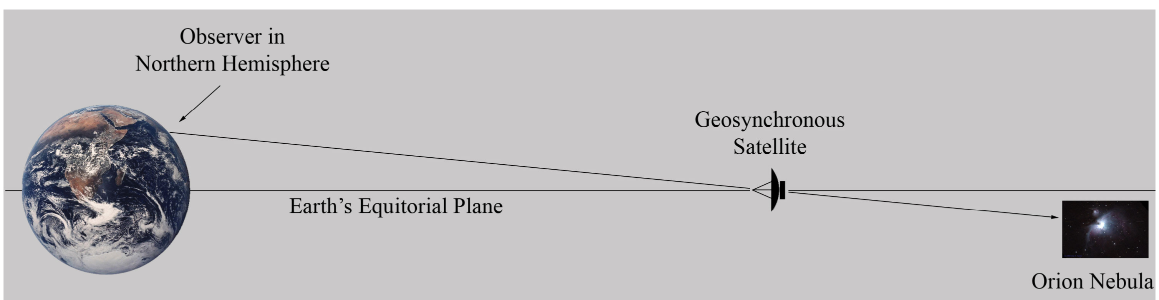
11/14/08

Introduction

When I first started doing astrophotography last January, it was only natural to take pictures of some of Orion's deep sky objects. Orion's Great Nebula, aka M42, is big, bright and easy to locate. So M42 was the perfect target to try to photograph when I hooked my DSLR to the telescope's prime focus. Since I didn't have my clock drive accurately polar aligned (it didn't have to be well aligned for visual work), I needed to use short exposures to minimize the trailing of stars. So I set my DSLR to 15 seconds for the exposure time and the scope (a 10'' Schmidt Newtonian) has a fast F ratio of 4, so with the camera's ISO bumped up to 3200, I could get reasonably sharp images although noisy. I had planned to combat the noise by taking a quantity of images and combining them in the computer so as to average out (reduce) the graininess. And this "stacking of images" actually works quite well. But upon examining some of the individual frames, I found west to east streaks across some of them, and they moved from frame to frame. See the following sample frame:

The sample frame shows three streaks which moved west to east (the direction was determined by comparing successive frames). North is up in the photo. At first I dismissed the streaks off as being some kind of space debris, but when I had taken more pictures of M42 on other nights, I would get more frames with the same type of west to east streaks. So I thought about the geometry of the situation and figured out I was actually imaging geostationary satellites which from my latitude appear to pass in front of M42! There's nothing like a little serendipity in science to get you to discover and explore something unexpected – at least unexpected to me when I was out taking pictures of the night sky.

The basic geometrical problem here is I'm in the northern hemisphere and I'm looking at an object that is south of the Earth's equatorial plane, so when I look at the object (M42 in my case) I'm looking through the equatorial plane and seeing geosynchronous satellites in front of M42. Since they move in synchronization with the Earth, they will appear as streaks when my telescope's clock drive effectively undoes the Earth's rotation. Most geosynchronous satellites orbit in the equatorial plane so as to appear stationary to an Earth based observer. These particular satellites are properly described as geostationary. The basic alignment, for my case, between the observer, the satellite and the Orion nebula is shown in the next diagram:



To be sure I was really understanding what I was observing, I thought I'd calculate the distance to such objects and see if the distance is anything like that published for the altitude for geostationary satellites. To be geostationary, a satellite's orbit needs to have a period of one mean solar day, be circular and be on the equatorial plane. If these conditions are met, a ground based antenna only needs to be pointed at the satellite once. I.e., the antenna doesn't have to do any tracking or other motion to stay aimed at the satellite. A common example is a rooftop satellite dish for TV.

Computation

To simplify the computations, we will assume this is a two dimensional problem. When the picture was taken, the Orion nebula was close to being due south from the observor's point of view. If the nebula were truly due south, then the center of the Earth, the observer, the satellite, and the nebula would all be on the same plane and the problem then becomes exactly two dimensional. When the nebula is not due south, then the observer is out of alignment due to the Earth's rotation. We will see that the error is not too big for this example.

First we need to know a little about the Earth's shape since I'm observing from its surface and not its center. We will use a standard reference geoid for the Earth. Basically the Earth's shape, at mean sea level, is an ellipse of revolution. This shape is also known as an oblate[1] spheroid. The important thing for the computation is the Earth in profile is an ellipse with the minor axis being the Earth's polar axis.

Earth's Reference Geoid Parameters	
Equitorial Radius (meters)	Polar Radius (meters)
$a = 6.378137 \times 10^6$	$b = 6.356752 \times 10^6$

From the venerable theory of ellipses, we define a numerical eccentricity as a function of the major and minor axes. It is:

$$\varepsilon = \sqrt{\frac{a^2 - b^2}{a^2}} = 0.081819791$$

This unitless number not only tells us the degree of circularity, but also turns out to be a common factor in equations describing ellipses. We will need it soon.

Since we are talking about the Earth's shape in profile, then we need two parameters about the observer's location, his latitude and altitude.

Observer's Positional Parameters	
Observer's Latitude (Degrees)	Observer's Altitude (Meters)
$\theta = 33.508^\circ$	$h = 200$

The observer in this case uses a GPS to find the location's parameters, but a topographic map will more than suffice for this job.

The distance from the center of the Earth to mean sea level at the observer's latitude is based on the well known formula for an ellipse. Since the Earth's major axis is the equatorial radius of the Earth, the angle, theta, is simply the latitude. Thus the distance is:

$$r = \frac{b}{\sqrt{1 - \varepsilon^2 \cos^2(\theta)}}$$

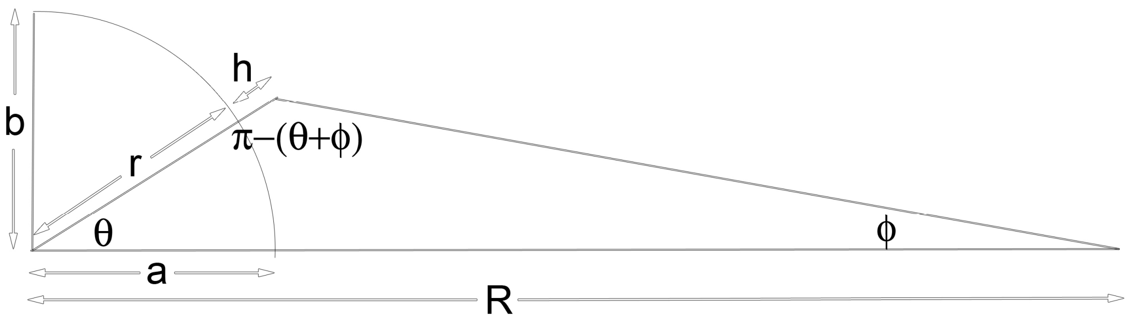
The observer's distance from the center of the Earth is then:

$$R_{Obs} = r + h$$

---

[1] A spheroid has three axes of symmetry. Oblate refers to the case where one axis is shorter than the other two that are the same. If the "odd man out" axis is longer, then the spheroid is prolate. In the case of the Earth, the short axis is through the poles and the other two axies are through the equator.

Now we are at the point of needing a diagram where we can see the appropriate variables and their relations. A couple of things will not be drawn to scale so as to make the diagram easier to read. For example, the observer's height above sea level is highly exaggerated and the distance to the satellite is drawn a bit too close to the Earth. In reality a geosynchronous satellite is about 6.7 Earth radii away from the Earth's center.



Since we wish to find "R", we will start with the "law of sines." It is for this diagram:

$$\frac{r+h}{\sin(\phi)} = \frac{R}{\sin(\pi-(\theta+\phi))}$$

Now we just solve for "R" and apply an appropriate trigonometric substitution, and we find:

$$R = (r+h)\big(\sin(\theta)\cot(\phi) + \cos(\theta)\big)$$

This can be further written as:

$$R = \left(\frac{b}{\sqrt{1-\varepsilon^2\cos^2(\theta)}} + h\right)\big(\sin(\theta)\cot(\phi) + \cos(\theta)\big)$$

Now with just one more piece of information, we can calculate the actual distance. The angle between the Orion nebula and the equatorial plane is call declination by astronomers, and the approximate declination of the nebula is -5.5 degrees. The negative sign means it is below the equatorial plane. So we will let phi be 5.5 degrees and we find:

$$R = 4.184424 \times 10^7 \text{ meters}$$

This is 26000 miles! If we now subtract the Earth's equatorial radius, we get 3.546611×10^7 meters, which is 22,038 miles above the equator. The published value for geostationary satellites is 22,236 miles giving our calculation a 0.9% relative error. So yes, I'm quite sure the objects in the photographs are geosynchronous satellites.

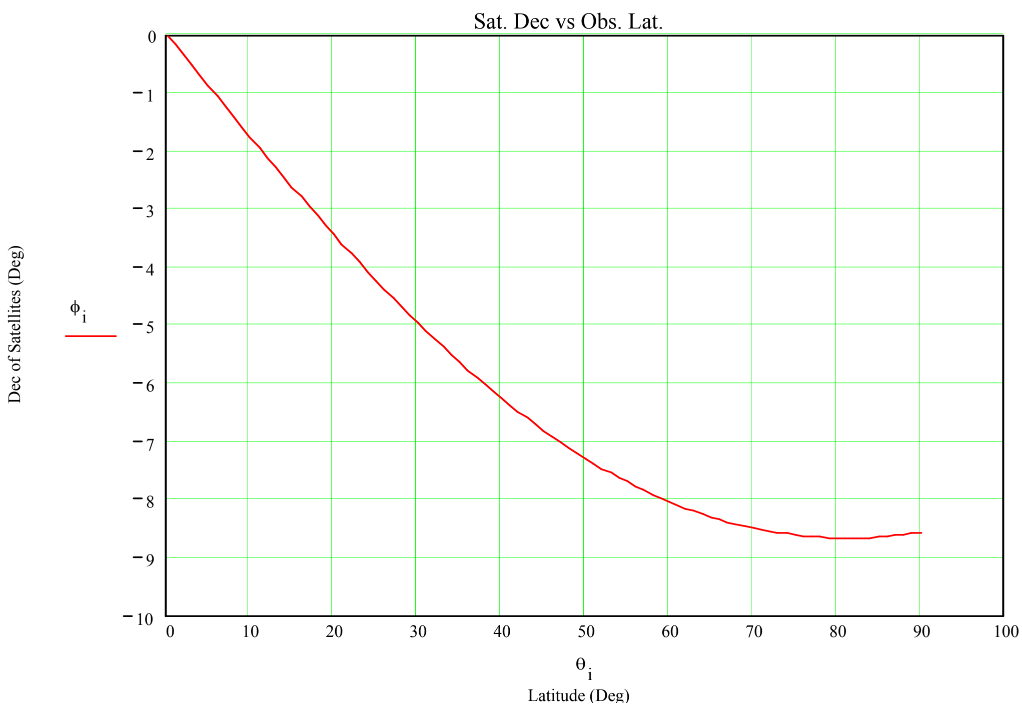
Now if we let “R” be the published distance to the satellites, we can reverse the formula to find out their observed declination (when on the meridian) given the observer’s latitude.

$$\phi = \tan^{-1}\left(\frac{r\sin(\theta)}{R - r\cos(\theta)}\right)$$

Where (we repeat here for convenience)

$$r = \frac{b}{\sqrt{1-\varepsilon^2\cos^2(\theta)}}$$

The following graph shows the result:



Since it is assumed the observer is in the northern hemisphere, the declinations are shown as negative meaning the satellites will appear south of the equatorial plane. For southern observers, the satellites will appear north (have positive declinations) of the equatorial plane.

It is interesting to note from the graph that the peak declination as a function of latitude is not at 90 degrees! As one move away from the equator the declination increases, but one also moves further away from the satellites which will decrease their declination! These two opposing effects cause a maximum declination to occur around 82 degrees of latitude.