Function for Legendre poly or order n

$$
\mathrm{Pn}(n,x) := \left|
\begin{array}{l}
\text{return } 1 \ \text{ if } n = 0 \\
\text{return } x \ \text{ if } n = 1 \\
x0 \leftarrow 1 \\
x1 \leftarrow x \\
\text{for } i \in 1..n-1 \\
\quad \left| \begin{array}{l}
xi \leftarrow \dfrac{(2 \cdot i + 1) \cdot x \cdot x1 - x0 \cdot i}{i + 1} \\
x0 \leftarrow x1 \\
x1 \leftarrow xi
\end{array} \right. \\
\text{return } x1
\end{array}
\right.
$$

Derivative of Legendre poly of order n

$$
\mathrm{Pnp}(n,x) := \left|
\begin{array}{l}
\text{return } 0 \ \text{ if } 0 = n \\
\text{return } \dfrac{n}{x^2 - 1} \cdot (x \cdot \mathrm{Pn}(n,x) - \mathrm{Pn}(n-1,x))
\end{array}
\right.
$$

Go calculate Gaussian Quad nodes and weights for Nth order fit on interval a to b

$$
\mathrm{BuildGauss}(N,a,b) := \left|
\begin{array}{l}
\text{for } j \in 0..N-1 \\
\quad \left| \begin{array}{l}
x \leftarrow \cos\left[\pi \cdot \dfrac{(4 \cdot (N - j) - 1)}{4 \cdot N + 2}\right] \\
dx \leftarrow 1 \\
\text{while } |dx| > 10^{-14} \\
\quad \left| \begin{array}{l}
dx \leftarrow \dfrac{\mathrm{Pn}(N,x)}{\mathrm{Pnp}(N,x)} \\
x \leftarrow x - dx
\end{array} \right. \\
y \leftarrow \mathrm{Pnp}(N,x) \\
G_{j,1} \leftarrow \dfrac{b - a}{\left(1 - x^2\right) \cdot y^2} \\
G_{j,0} \leftarrow (b - a) \cdot \dfrac{(x + 1)}{2} + a
\end{array} \right. \\
G
\end{array}
\right.
$$

$N := 200 \quad a := 0 \quad b := 10$

$G := \mathrm{BuildGauss}(N, a, b)$ Find 200 Gauss coefs from 0 to 10

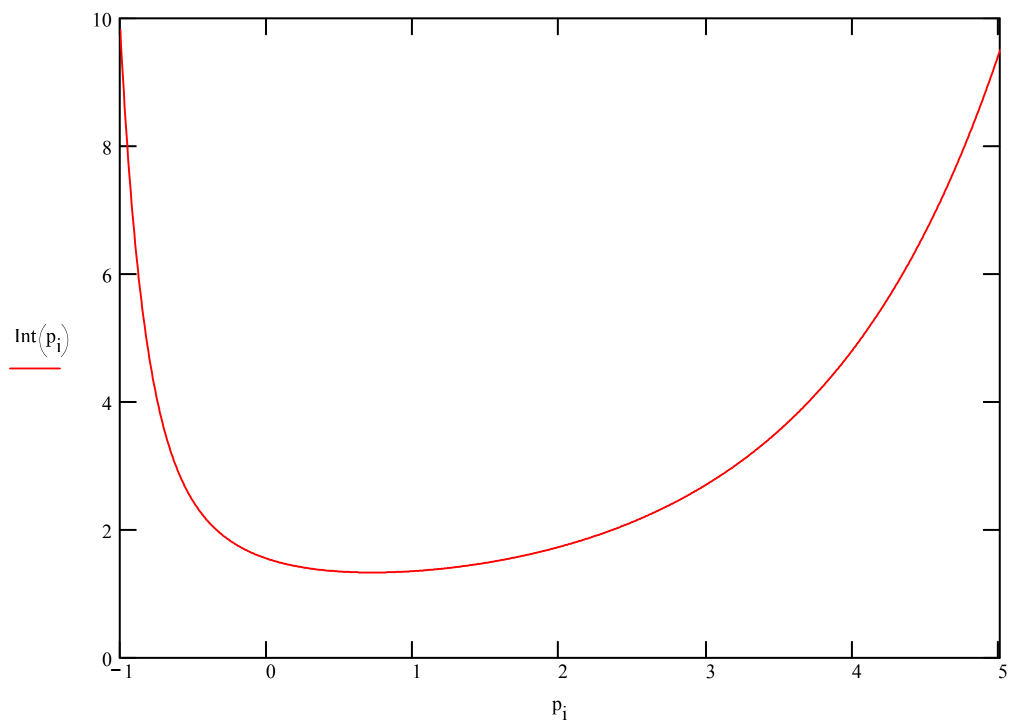
$I(p, x) := x^p \cdot e^{-x^2} \cdot I0(2 \cdot x)$ Function to integrate

$$\mathrm{Int}(p) := \sum_{i=0}^{N-1} G_{i,1} \cdot I\left(p, G_{i,0}\right)$$ Do the actual Gaussian Quadrature

$M := 1000 \quad plo := -0.999999 \quad phi := 5$

$i := 0 .. M$

$p_i := plo + (phi - plo) \cdot \frac{i}{M}$ Let p be on interval from -1 to 5 and look at graph



$\mathrm{Int}(0) = 1.5538993501 \quad \mathrm{Int}(1) = 1.3591409142 \quad \mathrm{Int}(2) = 1.7423093453 \quad \mathrm{Int}(3) = 2.7182818285$